

Low Energy Consequences of Five Dimensional $SO(10)$

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Abstract

We consider five dimensional (5D) supersymmetric $SO(10)$ compactified on the orbifold $S^1/(Z_2 \times Z'_2)$ such that the $SO(10)$ gauge symmetry is broken on both fixed points (branes), and the residual gauge symmetry is $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$. We explore one example in which the gauge symmetries on the two branes are respectively $SU(5) \times U(1)_X$ and $SU(4)_c \times SU(2)_L \times SU(2)_R$, and the MSSM gauge symmetry is recovered by the usual Higgs mechanism. We discuss how fermion masses and mixings can be understood in this framework by introducing a flavor $U(1)_F$ symmetry. Unification of the MSSM gauge couplings and proton stability are also considered. An order of magnitude increase in sensitivity could reveal proton decay.

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1 Introduction

Higher dimensional supersymmetric grand unified theories (SUSY GUTs) compactified on suitable orbifolds resolve the notorious doublet-triplet problem and eliminate the troublesome dimension five nucleon decay process associated with four dimensional (4D) SUSY GUTS in a relatively painless manner [1, 2, 3]. With the exception of just a handful of unified gauge groups such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ [4, 5], $SU(3)_c \times SU(3)_L \times SU(3)_R$ [6] and (flipped) $SU(5)' \times U(1)'_X$ [7] which allow rather elegant resolutions of these problems, most 4D SUSY GUTS require rather complicated Higgs systems and additional symmetries. Thus, one or more extra dimension(s) can play a crucial role in the construction of realistic models based on gauge groups $SU(5)$ [8], $SO(10)$ [9] and E_6 [10].

In this paper we investigate the construction and implications of five dimensional (5D) $SO(10)$ compactified on the orbifold $S^1/(Z_2 \times Z'_2)$ such that on each of the two fixed points (branes B1 and B2), the four dimensional gauge symmetry corresponds to one of the maximal subgroups of $SO(10)$ [11]. Thus, after compactification, the residual four dimensional gauge symmetry group is $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$. An additional $U(1)_X$ factor is present whose breaking is achieved via the standard Higgs mechanism. The MSSM gauge group is realized by spontaneously breaking $U(1)_X$ with a bulk Higgs hypermultiplet.

The plan of the paper is as follows. In section 2, 3 and 4 we describe the compactification scenario and the various symmetry breaking patterns from 5D $SO(10)$. Especially, in section 4 we discuss how the MSSM is realized at energies below M_c , with gauge symmetries $SU(5) \times U(1)$ and $SU(4)_c \times SU(2)_L \times SU(2)_R$ present on B1 and B2 respectively. Section 5 is devoted to a discussion of fermion masses and mixings, including the neutrino sector. We introduce a suitable $U(1)_F$ flavor symmetry, which enables us to realize the hierarchies displayed by the charged quark masses and mixings, as well as bilarge mixings in the neutrino sector. In section 6 we discuss the gauge coupling unification and proton stability based on the model of section 5. We

conclude in section 7.

2 Orbifold Symmetry Breakings in 5D $SO(10)$

The $SO(2n)$ generators are represented as $\begin{pmatrix} A+C & B+S \\ B-S & A-C \end{pmatrix}$, where A, B, C are $n \times n$ anti-symmetric matrices and S is an $n \times n$ symmetric matrix [12]. By an unitary transformation, the generators are given by

$$\begin{pmatrix} A-iS & C+iB \\ C-iB & A+iS \end{pmatrix}, \quad (1)$$

where A and S denote $U(n)$ generators, and $C \pm iB$ transform under $SU(n)$ as $n(n-1)/2$ and $\overline{n(n-1)/2}$, respectively. Under $SU(5) \times U(1)_X$, the $SO(10)$ generators are decomposed as

$$T_{SO(10)} = \left[\frac{\mathbf{24}_0 + \mathbf{1}_0}{\mathbf{10}_4} \middle| \frac{\mathbf{10}_{-4}}{\mathbf{24}_0 - \mathbf{1}_0} \right]_{10 \times 10}, \quad (2)$$

where the subscripts labeling the $SU(5)$ representations indicate $U(1)_X$ charges, and the subscript “ 10×10 ” denotes the matrix dimension. Also, $\mathbf{24}$ ($= \overline{\mathbf{24}}$) corresponds to $SU(5)$ generators, while $\text{diag}(\mathbf{1}_{5 \times 5}, -\mathbf{1}_{5 \times 5})$ is the $U(1)_X$ generator. The 5×5 matrices $\mathbf{24}_0$ and $\mathbf{10}_{-4}$ are further decomposed under $SU(3)_c \times SU(2)_L \times U(1)_Y$ as

$$\mathbf{24}_0 = \begin{pmatrix} (\mathbf{8}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{1})_0 & (\mathbf{3}, \overline{\mathbf{2}})_{-5/6} \\ (\overline{\mathbf{3}}, \mathbf{2})_{5/6} & (\mathbf{1}, \mathbf{3})_0 - (\mathbf{1}, \mathbf{1})_0 \end{pmatrix}_0, \quad (3)$$

$$\mathbf{10}_{-4} = \begin{pmatrix} (\overline{\mathbf{3}}, \mathbf{1})_{-2/3} & (\mathbf{3}, \mathbf{2})_{1/6} \\ (\mathbf{3}, \mathbf{2})_{1/6} & (\mathbf{1}, \mathbf{1})_1 \end{pmatrix}_{-4}. \quad (4)$$

Thus, each representation carries two independent $U(1)$ charges. Note that the two $(\mathbf{3}, \mathbf{2})_{1/6}$ s in $\mathbf{10}_{-4}$ are identified.

We intend to break $SO(10)$ to its maximal subgroups by Z_2 orbifoldings. Let us consider the action on $SO(10)$ of the following Z_2 group elements,

$$P_1 = \text{diag.} \left(+I_{3 \times 3}, +I_{2 \times 2}, +I_{3 \times 3}, +I_{2 \times 2} \right) \longrightarrow SO(10), \quad (5)$$

$$P_2 = \text{diag.} \left(+I_{3 \times 3}, +I_{2 \times 2}, -I_{3 \times 3}, -I_{2 \times 2} \right) \longrightarrow SU(5) \times U(1)_X, \quad (6)$$

$$P_3 = \text{diag.} \left(-I_{3 \times 3}, +I_{2 \times 2}, +I_{3 \times 3}, -I_{2 \times 2} \right) \longrightarrow SU(5)' \times U(1)'_X, \quad (7)$$

$$P_4 = \text{diag.} \left(+I_{3 \times 3}, -I_{2 \times 2}, +I_{3 \times 3}, -I_{2 \times 2} \right) \longrightarrow SU(4)_c \times SU(2)_L \times SU(2)_R, \quad (8)$$

where I 's denote identity matrices. Here the P 's all satisfy $P^2 = I_{5 \times 5}$. Eqs. (5)–(8) show all possible ways to define the 10 dimensional Z_2 group elements and the maximal subgroups of $SO(10)$ obtained by their operations, as will be explained below.

Under the operations $P_1 T_{SO(10)} P_1^{-1}$, $P_2 T_{SO(10)} P_2^{-1}$, \dots , the matrix elements of $T_{SO(10)}$ transform as

$$\left[\begin{array}{cc|cc} (\mathbf{8}, \mathbf{1})_0^{++++} & (\mathbf{3}, \mathbf{\bar{2}})_{-5/6}^{++--} & (\mathbf{\bar{3}}, \mathbf{1})_{-2/3}^{+--+} & (\mathbf{3}, \mathbf{2})_{1/6}^{+--+} \\ (\mathbf{\bar{3}}, \mathbf{2})_{5/6}^{++--} & (\mathbf{1}, \mathbf{3})_0^{++++} & (\mathbf{3}, \mathbf{2})_{1/6}^{+--+} & (\mathbf{1}, \mathbf{1})_1^{+--+} \\ \hline (\mathbf{3}, \mathbf{1})_{2/3}^{+--+} & (\mathbf{\bar{3}}, \mathbf{\bar{2}})_{-1/6}^{++--} & (\mathbf{8}, \mathbf{1})_0^{++++} & (\mathbf{\bar{3}}, \mathbf{2})_{5/6}^{+--+} \\ (\mathbf{\bar{3}}, \mathbf{\bar{2}})_{-1/6}^{++--} & (\mathbf{1}, \mathbf{1})_{-1}^{+--+} & (\mathbf{3}, \mathbf{\bar{2}})_{-5/6}^{++--} & (\mathbf{1}, \mathbf{3})_0^{++++} \end{array} \right]_{10 \times 10}, \quad (9)$$

where the superscripts of the matrix elements indicate the eigenvalues of P_1 , P_2 , P_3 , and P_4 respectively. Here, to avoid too much clutter, we have omitted the two $U(1)$ generators $((\mathbf{1}, \mathbf{1})_0^{++++})$. As shown in Eqs. (2) and (12), they appear in the diagonal part of the matrix (9).

For future convenience, let us define the $SO(10)$ generator pieces appearing in Eq. (9) more succinctly,

$$\left[\begin{array}{cc|cc} \mathbf{G}^{++++} & \mathbf{Q}'^{++--} & \mathbf{U}^{c+--+} & \mathbf{Q}^{+--+} \\ \mathbf{\bar{Q}}^{++--} & \mathbf{W}^{++++} & \mathbf{Q}^{+--+} & \mathbf{E}^{c+--+} \\ \hline \mathbf{\bar{U}}^{c+--+} & \mathbf{\bar{Q}}^{+--+} & \mathbf{G}^{++++} & \mathbf{\bar{Q}}'^{++--} \\ \mathbf{Q}^{+--+} & \mathbf{\bar{E}}^{c+--+} & \mathbf{Q}'^{++--} & \mathbf{W}^{++++} \end{array} \right], \quad (10)$$

whose entries are in one to one correspondence to those of Eq. (9). Note that \mathbf{Q} denotes $(\mathbf{3}, \mathbf{2})_{1/6}$, while \mathbf{Q}' denotes $(\mathbf{3}, \mathbf{\bar{2}})_{-5/6}$. Similarly, the two $U(1)$ generators $(\mathbf{1}, \mathbf{1})_0^{++++}$, which were omitted in Eq. (9), are defined as

$$\mathbf{Y}^{++++} \quad \text{and} \quad \mathbf{X}^{++++}, \quad (11)$$

where \mathbf{Y} corresponds to the hypercharge generator of SM. We identify the eigenvalues of the above generators with those of the associated gauge fields (and gauginos).

Suppose we have an $S^1/(Z_2 \times Z'_2)$ orbifold compactification in 5D space-time. The two Z_2 elements among Eqs. (5)–(8) can be employed so as to embed the internal

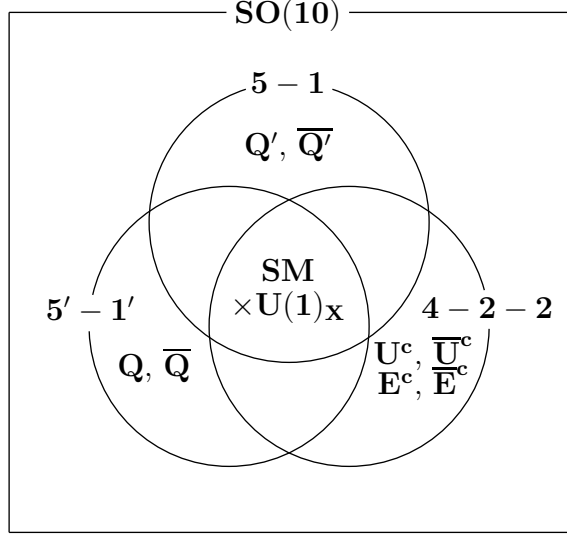


Figure 1: A diagram showing the generators of $SO(10)$ and its subgroups schematically. $5 - 1$, $5' - 1'$, $4 - 2 - 2$, and SM denote $SU(5) \times U(1)_X$, $SU(5)' \times U(1)'_X$, $SU(4)_c \times SU(2)_L \times SU(2)_R$, and the MSSM gauge group, respectively.

$Z_2 \times Z'_2$ into the two presumed reflection symmetries for the extra space, $y \leftrightarrow -y$ and $y' \leftrightarrow -y'$ ($y' = y + y_c/2$). Two eigenvalues of P_i could be interpreted as the parities (or boundary conditions) of the relevant fields under such reflections [13]. Thus, the wave function of a field with parity $(+-)$, for instance, must vanish on the brane at $y = y_c/2$ (B2), while it survives at $y = 0$ brane (B1). Only those fields assigned $(++)$ parities contain massless modes in their Kaluza-Klein (KK) spectrum. Thus, even though the bulk Lagrangian respects $SO(10)$, the effective low energy theory possesses a smaller gauge symmetry associated with the $(++)$ generators.

If P_1 (identity) and one more P_i ($i = 2, 3, 4$) are taken as $Z_2 \times Z'_2$ elements, the $SO(10)$ gauge symmetry breaks to $SU(5) \times U(1)_X$, $SU(5)' \times U(1)'_X$ [14], and $SU(4)_c \times SU(2)_L \times SU(2)_R$ [15, 16, 17], respectively. On the other hand, with two different P_i 's from among $\{P_2, P_3, P_4\}$, $SO(10)$ can be broken to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ [11, 18], as illustrated in Figure 1.

As is well known, compactification on $S^1/(Z_2 \times Z'_2)$ also can break the 4D $N = 2$

SUSY to $N = 1$. An $N = 2$ supersymmetric vector multiplet is split into an $N = 1$ vector multiplet and a chiral multiplet in adjoint representation. We assign the same parities as the generators to the associated vector multiplets as claimed above, but opposite parities to chiral multiplets. Then, $N = 2$ SUSY is broken to $N = 1$.

3 $SU(5) \times U(1)_X - SU(5)' \times U(1)'_X$

Let us consider the case in which P_2 and P_3 operations are chosen as Z_2 and Z'_2 elements [11], corresponding to the second and the third parities in Eq. (9) and (10). With P_2 , positive parities are assigned to the block-diagonal elements ($SU(5) \times U(1)_X$ generators and their associated gauge multiplets), while with P_3 , positive parities are assigned to the generators of $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ and \mathbf{Q}^{-+} , $\overline{\mathbf{Q}}^{-+}$, and to their associated gauge multiplets. Hence, after compactification, the gauge symmetry reduces to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$. Together with \mathbf{X}_V^{++} , the $SU(5)$ gauge multiplets in Eq. (10) survive at B1,

$$24_V = \left(\mathbf{G}_V^{++} + \mathbf{W}_V^{++} + \mathbf{Y}_V^{++} \right) + \left(\mathbf{Q}'_V^{+-} + \overline{\mathbf{Q}}_V^{+-} \right) \quad \text{at B1} , \quad (12)$$

where the subscripts V denote the vector multiplets. Thus $SU(5) \times U(1)_X$ should be preserved at B1 [13].

At B2 \mathbf{Q}'_V^{+-} and $\overline{\mathbf{Q}}_V^{+-}$ in Eq. (12) are replaced by \mathbf{Q}_V^{-+} and $\overline{\mathbf{Q}}_V^{-+}$, which are in the $\mathbf{10}_{-4}$ and $\overline{\mathbf{10}}_4$ representations of $SU(5) \times U(1)_X$,

$$24'_V = \left(\mathbf{G}_V^{++} + \mathbf{W}_V^{++} + \mathbf{Y}_V^{++} \right) + \left(\mathbf{Q}_V^{-+} + \overline{\mathbf{Q}}_V^{-+} \right) \quad \text{at B2} . \quad (13)$$

Note that the assigned hypercharges coincide with those given in ‘flipped’ $SU(5)' \times U(1)'_X$ [7]. The $U(1)'_X$ generator at B2, \mathbf{X}'^{++} is defined as

$$\text{diag}(\mathbf{1}_{3 \times 3}, -\mathbf{1}_{2 \times 2}, -\mathbf{1}_{3 \times 3}, \mathbf{1}_{2 \times 2}) . \quad (14)$$

Thus, the $U(1)'_X$ charges of the surviving elements at B2 turn out to be zero, while the other components are assigned -4 or 4 . The $U(1)'_X$ generator and the matrix

elements with $(++)$, $(-+)$ parities in Eq. (9) can be block-diagonalized to the form given in Eq. (2)

$$\left[\begin{array}{cc|cc} \mathbf{G}^{++} & \mathbf{Q}^{-+} & \mathbf{U}^{c--} & \mathbf{Q}'^{+-} \\ \overline{\mathbf{Q}}^{-+} & \mathbf{W}^{++} & \mathbf{Q}'^{+-} & \overline{\mathbf{E}}^{c--} \\ \hline \overline{\mathbf{U}}^{c--} & \overline{\mathbf{Q}}'^{+-} & \mathbf{G}^{++} & \overline{\mathbf{Q}}^{-+} \\ \overline{\mathbf{Q}}'^{+-} & \mathbf{E}^{c--} & \mathbf{Q}^{-+} & \mathbf{W}^{++} \end{array} \right], \quad (15)$$

through unitary transformation of the $SO(10)$ generator in Eq. (9) with

$$U_3 = \left(\begin{array}{c|ccc} I_{3 \times 3} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & I_{2 \times 2} \\ 0 & 0 & I_{3 \times 3} & 0 \\ 0 & I_{2 \times 2} & 0 & 0 \end{array} \right)_{10 \times 10}. \quad (16)$$

In Eq. (15), the two superscripts denote the eigenvalues of P_2 and P_3 . From Eq. (15), we conclude that the gauge symmetry at B2 is associated with a second (flipped) $SU(5)' \times U(1)'_X$ embedded in $SO(10)$ [7].

To break 4D $N = 2$ SUSY, opposite parities should be assigned to the chiral multiplet $(\Phi + iA_5, \lambda_2)$, where Φ , A_5 , λ_2 belong to $N = 2$ vector multiplets. The non-vanishing chiral multiplets at B1 are

$$\mathbf{10}_\Sigma = \mathbf{U}_\Sigma^{c++} + \mathbf{E}_\Sigma^{c++} + \mathbf{Q}_\Sigma^{+-}, \quad (17)$$

$$\overline{\mathbf{10}}_\Sigma = \overline{\mathbf{U}}_\Sigma^{c++} + \overline{\mathbf{E}}_\Sigma^{c++} + \overline{\mathbf{Q}}_\Sigma^{+-}, \quad (18)$$

while on B2, \mathbf{Q}_Σ^{+-} and $\overline{\mathbf{Q}}_\Sigma^{+-}$ are replaced by \mathbf{Q}'_Σ^{+-} and $\overline{\mathbf{Q}}'_\Sigma^{+-}$ (which are in $\mathbf{24}_\Sigma$ and $\mathbf{24}'_\Sigma$ at B1). Together with the vector-like pairs with $(++)$ parities, they compose $\mathbf{10}'_{-4}$ and $\overline{\mathbf{10}}'_4$ -plets of $SU(5)' \times U(1)'_X$ at B2,

$$\mathbf{10}'_\Sigma = \mathbf{U}_\Sigma^{c++} + \overline{\mathbf{E}}_\Sigma^{c++} + \mathbf{Q}'_\Sigma^{-+}, \quad (19)$$

$$\overline{\mathbf{10}}'_\Sigma = \overline{\mathbf{U}}_\Sigma^{c++} + \mathbf{E}_\Sigma^{c++} + \overline{\mathbf{Q}}'_\Sigma^{-+}. \quad (20)$$

We note in Eqs. (17)–(20) the appearance of two vector-like pairs \mathbf{U}_Σ^{c++} , $\overline{\mathbf{U}}_\Sigma^{c++}$ and \mathbf{E}_Σ^{c++} , $\overline{\mathbf{E}}_\Sigma^{c++}$, which contain massless modes. We summarize the above results in Table I.

Vector (B1)	$\mathbf{24}_V, \mathbf{1}_V$	$\mathbf{G}_V^{++}, \mathbf{W}_V^{++}, \mathbf{Y}_V^{++}, \mathbf{X}_V^{++}, \mathbf{Q}_V'^{+-}, \mathbf{Q}_V'^{+-}$
Chiral (B1)	$\mathbf{10}_\Sigma, \overline{\mathbf{10}}_\Sigma$	$\mathbf{U}_\Sigma^{c++}, \mathbf{E}_\Sigma^{c++}, \mathbf{Q}_\Sigma^{+-}, \overline{\mathbf{U}}_\Sigma^{c++}, \overline{\mathbf{E}}_\Sigma^{c++}, \overline{\mathbf{Q}}_\Sigma^{+-}$
Vector (B2)	$\mathbf{24}'_V, \mathbf{1}'_V$	$\mathbf{G}_V^{++}, \mathbf{W}_V^{++}, \mathbf{Y}_V^{++}, \mathbf{X}_V'^{++}, \mathbf{Q}_V^{+-}, \overline{\mathbf{Q}}_V'^{+-}$
Chiral (B2)	$\mathbf{10}'_\Sigma, \overline{\mathbf{10}}'_\Sigma$	$\mathbf{U}_\Sigma^{c++}, \overline{\mathbf{E}}_\Sigma^{c++}, \mathbf{Q}_\Sigma'^{+-}, \overline{\mathbf{U}}_\Sigma^{c++}, \mathbf{E}_\Sigma^{c++}, \overline{\mathbf{Q}}_\Sigma'^{+-}$

Table I. Surviving superfields on each brane in the $SO(10)$ gauge multiplet.

To preserve the successful MSSM gauge coupling unification, we need to remove them from the low energy spectrum. To realize the MSSM gauge symmetry at lower energies, we employ the Higgs mechanism via bulk Higgs fields. This is because with brane Higgs fields, it is hard to provide heavy masses for the vector-like pairs, \mathbf{U}_Σ^{c++} , $\overline{\mathbf{U}}_\Sigma^{c++}$, and \mathbf{E}_Σ^{c++} , $\overline{\mathbf{E}}_\Sigma^{c++}$. Let us introduce two pairs of Higgs hypermultiplets $\mathbf{16}, \overline{\mathbf{16}}$ as shown in Table II.

Hypermultiplets	$Z_2 \times Z'_2$ parities	$U(1)_R$
$\mathbf{16}_H$	$\mathbf{u}^{c--}, \mathbf{e}^{c--}, \mathbf{q}^{-+}; \mathbf{d}^{c++}, \mathbf{l}^{+-}; \nu^{c++}$	0
$\mathbf{16}^c_H$	$\mathbf{u}^{++}, \mathbf{e}^{++}, \mathbf{q}^{c+-}; \mathbf{d}^{--}, \mathbf{l}^{c-+}; \nu^{--}$	0
$\overline{\mathbf{16}}_H$	$\overline{\mathbf{u}}^{c--}, \overline{\mathbf{e}}^{c--}, \overline{\mathbf{q}}^{-+}; \overline{\mathbf{d}}^{c++}, \overline{\mathbf{l}}^{+-}; \overline{\nu}^{c++}$	0
$\overline{\mathbf{16}}^c_H$	$\overline{\mathbf{u}}^{++}, \overline{\mathbf{e}}^{++}, \overline{\mathbf{q}}^{c+-}; \overline{\mathbf{d}}^{--}, \overline{\mathbf{l}}^{c-+}; \overline{\nu}^{--}$	0

Table II. $Z_2 \times Z'_2$ parities of the bulk Higgs hypermultiplets.

From $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$, the surviving fields at B1 and B2 are

$$\mathbf{16}_H : (\mathbf{d}^{c++}, \mathbf{l}^{+-}; \nu^{c++}) \text{ at B1 ,} \quad (21)$$

$$(\mathbf{d}^{c++}, \mathbf{q}^{-+}, \nu^{c++}) \text{ at B2 ,} \quad (22)$$

$$\overline{\mathbf{16}}_H : (\overline{\mathbf{d}}^{c++}, \overline{\mathbf{l}}^{+-}; \overline{\nu}^{c++}) \text{ at B1 ,} \quad (23)$$

$$(\overline{\mathbf{d}}^{c++}, \overline{\mathbf{q}}^{-+}, \overline{\nu}^{c++}) \text{ at B2 ,} \quad (24)$$

They compose $(\overline{\mathbf{5}}_{-3}; \mathbf{1}_5)$ and $(\mathbf{5}_3; \overline{\mathbf{1}}_{-5})$ representations of $SU(5) \times U(1)_X$ at B1, and $\mathbf{10}'_1, \overline{\mathbf{10}}'_{-1}$ of $SU(5)' \times U(1)'_X$ at B2.

In order to realize $N = 1$ SUSY, the surviving fields from $\mathbf{16}^c, \overline{\mathbf{16}}^c$ on the two branes should be as follows:

$$\mathbf{16}^c_H : (\mathbf{u}^{++}, \mathbf{q}^{c+-}, \mathbf{e}^{++}) \text{ at B1 ,} \quad (25)$$

$$: (\mathbf{u}^{++}, \mathbf{l}^{c-+}; \mathbf{e}^{++}) \quad \text{at B2} , \quad (26)$$

$$\overline{\mathbf{16}}^c_H : (\overline{\mathbf{u}}^{++}, \overline{\mathbf{q}}^{c+-}, \overline{\mathbf{e}}^{++}) \quad \text{at B1} , \quad (27)$$

$$: (\overline{\mathbf{u}}^{++}, \overline{\mathbf{l}}^{c-+}; \overline{\mathbf{e}}^{++}) \quad \text{at B2} . \quad (28)$$

They compose $\mathbf{10}^c_{-1}$, $\overline{\mathbf{10}}^c_1 (= \mathbf{10}_1)$ at B1, and $(\overline{\mathbf{5}}^{c'}_3; \mathbf{1}^{c'}_{-5})$, $(\mathbf{5}^{c'}_{-3}; \overline{\mathbf{1}}^c_5)$ at B2, respectively. The results are summarized in Table III.

B1	$\mathbf{5}_H, \mathbf{1}_H$	$\overline{\mathbf{5}}_H, \overline{\mathbf{1}}_H$	$\mathbf{10}^c_H$	$\overline{\mathbf{10}}^c_H$
	$\mathbf{d}^{c++}, \mathbf{l}^{+-}, \nu^{c++}$	$\overline{\mathbf{d}}^{c++}, \overline{\mathbf{l}}^{+-}, \overline{\nu}^{c++}$	$\mathbf{u}^{++}, \mathbf{e}^{++}, \mathbf{q}^{c+-}$	$\overline{\mathbf{u}}^{++}, \overline{\mathbf{e}}^{++}, \overline{\mathbf{q}}^{c+-}$
B2	$\mathbf{10}'_H$	$\overline{\mathbf{10}}'_H$	$\mathbf{5}^{c'}_H, \mathbf{1}^{c'}_H$	$\overline{\mathbf{5}}^{c'}_H, \overline{\mathbf{1}}^{c'}_H$
	$\mathbf{d}^{c++}, \nu^{c++}, \mathbf{q}^{-+}$	$\overline{\mathbf{d}}^{c++}, \overline{\nu}^{c++}, \overline{\mathbf{q}}^{-+}$	$\mathbf{u}^{++}, \mathbf{l}^{c-+}, \mathbf{e}^{++}$	$\overline{\mathbf{u}}^{++}, \overline{\mathbf{l}}^{c-+}, \overline{\mathbf{e}}^{++}$

Table III. Surviving Higgs superfields on the branes B1 and B2.

Consider the following Higgs superpotentials on the two branes

$$W_{B1} = \kappa_1 S \left(\mathbf{16}_H \overline{\mathbf{16}}_H - M_1^2 \right) , \quad (29)$$

$$W_{B2} = \kappa_2 S \left(\mathbf{16}'_H \overline{\mathbf{16}}'_H + \mathbf{11}' - M_2^2 \right) , \quad (30)$$

where $\kappa_{1,2}$ ($M_{1,2}$) are dimensionless (dimensionful) parameters. Here $\mathbf{16}_H \overline{\mathbf{16}}_H$ stands for the superpotential couplings by the surviving Higgs at B1 shown in Table III, $\mathbf{10}^c_H \overline{\mathbf{10}}^c_H + \overline{\mathbf{5}}_H \mathbf{5}_H + \mathbf{1}_H \overline{\mathbf{1}}_H$ with arbitrary coefficients. $\mathbf{16}'_H \overline{\mathbf{16}}'_H$ in Eq. (30) is also similarly understood. S is a bulk singlet superfield with unit $U(1)_R$ charge, which can couple to the Higgs fields on both branes. Also, $\mathbf{1}$, $\mathbf{1}'$ are gauge singlet fields with suitable $U(1)_R$ charges. With non-zero vacuum expectation values (VEVs) of the scalar components of ν^{c++} , $\overline{\nu}^{c++}$, $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ is spontaneously broken to the MSSM gauge group. Note that suitable VEVs of $\mathbf{1}$, $\mathbf{1}'$ can ensure that the VEVs $\langle \nu^{c++} \rangle$ and $\langle \overline{\nu}^{c++} \rangle$ are constant along the extra dimension.

With spontaneous symmetry breaking, the gauge bosons, gauge scalars and their superpartners in $\mathbf{10}_{-4}$, $\overline{\mathbf{10}}_4$ acquire masses. The gauge bosons in \mathbf{U}^{c--}_V , \mathbf{Q}^{c+}_V , \mathbf{E}^{c--}_V , and $\overline{\mathbf{U}}^{c--}_V$, $\overline{\mathbf{Q}}^{c+}_V$, $\overline{\mathbf{E}}^{c--}_V$ absorb a linear combination of A_5 's from

$$\mathbf{U}^{c++}_\Sigma (n \neq 0) , \quad \mathbf{Q}^{+-}_\Sigma , \quad \mathbf{E}^{c++}_\Sigma (n \neq 0) , \quad \text{and} \quad (31)$$

$$\overline{\mathbf{U}}^{c++}_\Sigma (n \neq 0) , \quad \overline{\mathbf{Q}}^{+-}_\Sigma , \quad \overline{\mathbf{E}}^{c++}_\Sigma (n \neq 0) , \quad (32)$$

and from the Higgs fields

$$\mathbf{u}^{\mathbf{c}^{--}} , \quad \mathbf{q}^{-+} , \quad \mathbf{e}^{\mathbf{c}^{--}} , \quad \text{and} \quad (33)$$

$$\overline{\mathbf{u}}^{\mathbf{c}^{--}} , \quad \overline{\mathbf{q}}^{-+} , \quad \overline{\mathbf{e}}^{\mathbf{c}^{--}} . \quad (34)$$

The massless ($n = 0$) modes of the gauge scalars Φ , A_5 s in $\mathbf{U}_{\Sigma}^{\mathbf{c}^{++}}$, $\mathbf{E}_{\Sigma}^{\mathbf{c}^{++}}$, and $\overline{\mathbf{U}}_{\Sigma}^{\mathbf{c}^{++}}$, $\overline{\mathbf{E}}_{\Sigma}^{\mathbf{c}^{++}}$ obtain masses from the gauge coupling $g^2 |\langle \nu_H^c \rangle A_5|^2$, where ν_H^c (ν_H^{c*}) is the scalar component of $\nu^{\mathbf{c}^{++}}$, $\overline{\nu}^{\mathbf{c}^{++}}$. The gauge bosons in \mathbf{Q}'_V^{+-} and $\overline{\mathbf{Q}}'_V^{+-}$ absorb the A_5 's from

$$\mathbf{Q}'_{\Sigma}^{-+} , \quad \overline{\mathbf{Q}}'_{\Sigma}^{-+} . \quad (35)$$

We note that the gauge bosons absorb A_5 's carrying the same quantum numbers but opposite parities, whereas they absorb the Higgs fields with the same parities. This can be understood from the Lagrangian after symmetry breaking, $\mathcal{L} \supset (\partial_5 A_{\mu} - \partial_{\mu} A_5)^2 \sim m_{KK}^2 (A_{\mu} - \frac{1}{m_{KK}} \partial_{\mu} A_5)^2$ and $\mathcal{L} \supset g^2 v^2 (A_{\mu} - \frac{1}{gv} \partial_{\mu} a)^2$, where m_{KK} indicates the KK mass and a is the Goldstone boson of the scalar Higgs $\phi = (v + \rho) e^{ia/v} / \sqrt{2}$.

Finally, in order to realize the MSSM field contents at low energies, we should ensure that the three vector-like pairs of Higgs fields, \mathbf{u}^{++} , $\overline{\mathbf{u}}^{++}$, $\mathbf{d}^{\mathbf{c}^{++}}$, $\overline{\mathbf{d}}^{\mathbf{c}^{++}}$, and \mathbf{e}^{++} , $\overline{\mathbf{e}}^{++}$ are heavy. This is possible, for example, by introducing at B1 additional brane chiral superfields $\mathbf{10}_{-1}^b$, $\overline{\mathbf{10}}_1^b$, and $\overline{\mathbf{5}}_{-3}^b$, $\mathbf{5}_3^b$ with unit $U(1)_R$ charges. (Gauge symmetry forbids their couplings to the chiral multiplets from the 4D $N = 2$ vector multiplet.)

4 $SU(5) \times U(1)_X - SU(4)_c \times SU(2)_L \times SU(2)_R$

In this section, we take $Z_2 \times Z'_2$ elements to be the P_2 and P_4 . As already explained, by a P_2 operation, the $SU(5) \times U(1)_X$ generators are assigned positive parities and their associated gauge multiplets survive at B1. On the other hand, the $SO(10)$ generators with even parity under P_4 are

$$(\mathbf{8}, \mathbf{1})_0^{++} , \quad (\overline{\mathbf{3}}, \mathbf{1})_{-2/3}^{-+} , \quad (\mathbf{3}, \mathbf{1})_{2/3}^{-+} , \quad (\mathbf{1}, \mathbf{1})_0^{++} ; \quad (36)$$

$$(\mathbf{1}, \mathbf{3})_0^{++} , \quad (\mathbf{1}, \mathbf{1})_1^{-+} , \quad (\mathbf{1}, \mathbf{1})_{-1}^{-+} , \quad (\mathbf{1}, \mathbf{1})_0^{++} , \quad (37)$$

all of which survive at B2. Here the superscripts denote P_2 and P_4 eigenvalues. The generators in Eqs. (36) and (37) correspond to $SO(6)$ and $SO(4)$, respectively. To see this explicitly, we transform the $SO(10)$ generator in Eq. (9) with the unitary matrix,

$$U_4 = \left(\begin{array}{c|cc|c} I_{3 \times 3} & 0 & 0 & 0 \\ \hline 0 & 0 & I_{2 \times 2} & 0 \\ 0 & I_{3 \times 3} & 0 & 0 \\ \hline 0 & 0 & 0 & I_{2 \times 2} \end{array} \right)_{10 \times 10}. \quad (38)$$

The entries with even parities under P_4 are then block-diagonalized,

$$\left[\begin{array}{cc|cc} (\mathbf{8}, \mathbf{1})_0^{++} & (\mathbf{\bar{3}}, \mathbf{1})_{-2/3}^{-+} & (\mathbf{3}, \mathbf{\bar{2}})_{-5/6}^{+-} & (\mathbf{3}, \mathbf{2})_{1/6}^{--} \\ (\mathbf{3}, \mathbf{1})_{2/3}^{-+} & (\mathbf{8}, \mathbf{1})_0^{++} & (\mathbf{\bar{3}}, \mathbf{\bar{2}})_{-1/6}^{--} & (\mathbf{\bar{3}}, \mathbf{2})_{5/6}^{+-} \\ \hline (\mathbf{\bar{3}}, \mathbf{2})_{5/6}^{+-} & (\mathbf{3}, \mathbf{2})_{1/6}^{--} & (\mathbf{1}, \mathbf{3})_0^{++} & (\mathbf{1}, \mathbf{1})_1^{-+} \\ (\mathbf{\bar{3}}, \mathbf{\bar{2}})_{-1/6}^{--} & (\mathbf{3}, \mathbf{\bar{2}})_{-5/6}^{+-} & (\mathbf{1}, \mathbf{1})_{-1}^{-+} & (\mathbf{1}, \mathbf{3})_0^{++} \end{array} \right]_{10 \times 10}, \quad (39)$$

where we have omitted the two $U(1)$ generators $((\mathbf{1}, \mathbf{1})_0^{++})$ s from the diagonal parts. Using Eq. (1), one can readily check that the two block-diagonal parts are $SO(6) \times SO(4)$ ($\sim SU(4)_c \times SU(2)_L \times SU(2)_R$) generators. The two off diagonal parts in Eq. (39) are identified with each other, and they compose the $(\mathbf{6}, \mathbf{2}, \mathbf{2})$ representations under $SU(4)_L \times SU(2)_L \times SU(2)_R$. We conclude that by employing P_2 and P_4 , $SU(5) \times U(1)_X$ and $SU(4)_L \times SU(2)_L \times SU(2)_R$ are preserved at B1 and B2, respectively. The parities of $N = 1$ gauge multiplets follow those of the corresponding generators.

With opposite parities assigned to the chiral multiplets, the non-vanishing components at B1 are

$$\mathbf{10}_\Sigma = \mathbf{U}_\Sigma^{c+-} + \mathbf{E}_\Sigma^{c+-} + \mathbf{Q}_\Sigma^{++}, \quad \text{and} \quad (40)$$

$$\overline{\mathbf{10}}_\Sigma = \overline{\mathbf{U}}_\Sigma^{c+-} + \overline{\mathbf{E}}_\Sigma^{c+-} + \overline{\mathbf{Q}}_\Sigma^{++}, \quad (41)$$

while, on B2 brane, the surviving chiral multiplet is

$$(\mathbf{6}, \mathbf{2}, \mathbf{2})_\Sigma = \mathbf{Q}_\Sigma^{++} + \overline{\mathbf{Q}}_\Sigma^{++} + \mathbf{Q}'_\Sigma{}^{-+} + \overline{\mathbf{Q}'}_\Sigma{}^{-+}. \quad (42)$$

Here we used the notations from Eq. (10), and the subscript “ Σ ” stands for the chiral multiplet. We show in Table IV the surviving vector and chiral multiplets on each brane.

Vector (B1)	$\mathbf{24}_V, \mathbf{1}_V$	$\mathbf{G}_V^{++}, \mathbf{W}_V^{++}, \mathbf{Y}_V^{++}; \mathbf{X}_V^{++}, \mathbf{Q}_V^{+-}, \mathbf{Q}_V^{\prime+-}$
Chiral (B1)	$\mathbf{10}_\Sigma, \overline{\mathbf{10}}_\Sigma$	$\mathbf{U}_\Sigma^{c+-}, \mathbf{E}_\Sigma^{c+-}, \mathbf{Q}_\Sigma^{++}; \overline{\mathbf{U}}_\Sigma^{c+-}, \overline{\mathbf{E}}_\Sigma^{c+-}, \overline{\mathbf{Q}}_\Sigma^{++}$
Vector (B2)	$\mathbf{15}_V, \mathbf{3}_V, \mathbf{3}'_V$	$\mathbf{G}_V^{++}, \mathbf{U}_V^{c-+}, \overline{\mathbf{U}}_V^{c-+}, \mathbf{X}_V^{++}; \mathbf{W}_V^{++}; \mathbf{E}_V^{c-+}, \overline{\mathbf{E}}_V^{c-+}, \mathbf{Y}_V^{++}$
Chiral (B2)	$(\mathbf{6}, \mathbf{2}, \mathbf{2})_\Sigma$	$\mathbf{Q}_\Sigma^{++}, \overline{\mathbf{Q}}_\Sigma^{++}, \mathbf{Q}'_\Sigma^{++}, \overline{\mathbf{Q}}'_\Sigma^{++}$

Table IV. Surviving superfields on each brane in the $SO(10)$ gauge multiplet.

As seen from Table IV, the vector-like pair, \mathbf{Q}_Σ^{++} and $\overline{\mathbf{Q}}_\Sigma^{++}$ must be removed from the low energy spectrum. They can become massive through spontaneous symmetry breaking by the bulk Higgs. Table V shows the Higgs hypermultiplets and their quantum numbers.

Hypermultiplets	$Z_2 \times Z'_2$ parities	$U(1)_R$
$\mathbf{16}_H$	$\mathbf{u}^{c-+}, \mathbf{e}^{c-+}, \mathbf{q}^{--}; \mathbf{d}^{c++}, \mathbf{l}^{+-}; \nu^{c++}$	0
$\mathbf{16}^c_H$	$\mathbf{u}^{+-}, \mathbf{e}^{+-}, \mathbf{q}^{c++}; \mathbf{d}^{--}, \mathbf{l}^{c-+}; \nu^{--}$	0
$\overline{\mathbf{16}}_H$	$\overline{\mathbf{u}}^{c-+}, \overline{\mathbf{e}}^{c-+}, \overline{\mathbf{q}}^{--}; \overline{\mathbf{d}}^{c++}, \overline{\mathbf{l}}^{+-}; \overline{\nu}^{c++}$	0
$\overline{\mathbf{16}}^c_H$	$\overline{\mathbf{u}}^{+-}, \overline{\mathbf{e}}^{+-}, \overline{\mathbf{q}}^{c++}; \overline{\mathbf{d}}^{--}, \overline{\mathbf{l}}^{c-+}; \overline{\nu}^{--}$	0

Table V. $Z_2 \times Z'_2$ parities of the bulk Higgs hypermultiplets.

Analogous to the previous case with $SU(5) \times U(1)_X - SU(5)' \times U(1)'_X$, at B1 they compose $SU(5) \times U(1)_X$ multiplets, $\mathbf{10}_1, \overline{\mathbf{5}}_{-3}, \mathbf{1}_5$, etc. At B2 they compose $SU(4)_c \times SU(2)_L \times SU(2)_R$ multiplets such as $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ as shown in Table VI.

$\mathbf{5}_H, \mathbf{1}_H$ (B1)	$\overline{\mathbf{5}}_H, \overline{\mathbf{1}}_H$ (B1)	$\mathbf{10}^c_H$ (B1)	$\overline{\mathbf{10}}^c_H$ (B1)
$\mathbf{d}^{c++}, \mathbf{l}^{+-}, \nu^{c++}$	$\overline{\mathbf{d}}^{c++}, \overline{\mathbf{l}}^{+-}, \overline{\nu}^{c++}$	$\mathbf{u}^{+-}, \mathbf{e}^{+-}, \mathbf{q}^{c++}$	$\overline{\mathbf{u}}^{+-}, \overline{\mathbf{e}}^{+-}, \overline{\mathbf{q}}^{c++}$
$(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_H$ (B2)	$(\mathbf{4}, \mathbf{1}, \mathbf{2})_H$ (B2)	$(\mathbf{4}^c, \mathbf{2}, \mathbf{1})_H$ (B2)	$(\overline{\mathbf{4}}^c, \mathbf{2}, \mathbf{1})_H$ (B2)
$\mathbf{u}^{c-+}, \mathbf{e}^{c-+}, \mathbf{d}^{c++}, \nu^{c++}$	$\overline{\mathbf{u}}^{c-+}, \overline{\mathbf{e}}^{c-+}, \overline{\mathbf{d}}^{c++}, \overline{\nu}^{c++}$	$\mathbf{q}^{c++}, \mathbf{l}^{c-+}$	$\overline{\mathbf{q}}^{c++}, \overline{\mathbf{l}}^{c-+}$

Table VI. Surviving Higgs superfields on the branes B1 and B2.

On the two branes, the Higgs superpotentials are

$$W_{B1} = \kappa_1 S \left(\mathbf{16}_H \overline{\mathbf{16}}_H - M_1^2 \right), \quad (43)$$

$$W_{B2} = \kappa_2 S \left(\mathbf{16}^c_H \overline{\mathbf{16}}^c_H + \mathbf{11}' - M_2^2 \right), \quad (44)$$

where we schematically wrote the vector-like couplings of the Higgs multiplets on the two branes, $\mathbf{10}_H^c \overline{\mathbf{10}}_H^c + \overline{\mathbf{5}}_H \mathbf{5}_H + \mathbf{1}_H \overline{\mathbf{1}}_H$ and $(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_H (\mathbf{4}, \mathbf{1}, \mathbf{2})_H + (\mathbf{4}^c, \mathbf{2}, \mathbf{1})_H (\overline{\mathbf{4}}^c, \mathbf{2}, \mathbf{1})_H$ with arbitrary coefficients as $\mathbf{16}_H \overline{\mathbf{16}}_H$ and $\mathbf{16}_H^c \overline{\mathbf{16}}_H^c$, respectively. The gauge singlet superfields $\mathbf{1}, \mathbf{1}'$ are introduced for the same reason as in section 3. As in the previous case, the VEVs of $\nu^{c++}, \overline{\nu}^{c++}$ lead to the MSSM gauge symmetry, and generate mass terms of $\mathbf{X}_V^{++}, \mathbf{Q}_\Sigma^{++}$, and $\overline{\mathbf{Q}}_\Sigma^{++}$. Additional B1 brane superfields $\mathbf{10}_1^b, \overline{\mathbf{10}}_{-1}^b$, and $\overline{\mathbf{5}}_{-3}^b, \mathbf{5}_3^b$ with $U(1)_R$ charges of unity, and their bilinear couplings with the Higgs fields at B1 could simply make $\mathbf{d}^{c++}, \overline{\mathbf{d}}^{c++}, \mathbf{q}^{c++}, \overline{\mathbf{q}}^{c++}$, etc. heavy.

Finally another scenario one could consider is one with $SU(5)' \times U(1)'_X$ and $SU(4)_c \times SU(2)_L \times SU(2)_R$ at B1 and B2 respectively. We will not pursue this any further here.

5 Fermion Masses and Mixings

Let us consider the $SU(5) \times U(1)_X - SU(4)_c \times SU(2)_L \times SU(2)_R$ model. We place the second and third generation quarks and leptons on $SU(5) \times U(1)_X$ brane (B1), and the first generation on $SU(4)_c \times SU(2)_L \times SU(2)_R$ brane (B2). We also introduce an $U(1)_F$ flavor symmetry and a singlet bulk flavon field ‘ F ’ carrying $U(1)_F$ charge of -1 , such that $\frac{\langle F \rangle}{M_*} = \epsilon \approx 0.2$, where M_* denotes the fundamental scale. The $U(1)_F$ charge assignments of the MSSM fields are shown in Table VII.

Representation	Family	Fields	$U(1)_R$	$U(1)_F$
$\mathbf{10}_1$	3rd	$\mathbf{u}_3^c, \mathbf{e}_3^c, \mathbf{q}_3$	1/2	0
(B1)	2nd	$\mathbf{u}_2^c, \mathbf{e}_2^c, \mathbf{q}_2$	1/2	2
$\overline{\mathbf{5}}_{-3}$	3rd	$\mathbf{d}_3^c, \mathbf{l}_3$	1/2	3
(B1)	2nd	$\mathbf{d}_2^c, \mathbf{l}_2$	1/2	3
$\mathbf{1}_5$	3rd	ν_3^c	1/2	—
(B1)	2nd	ν_2^c	1/2	—
$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	1st	$\mathbf{q}_1, \mathbf{l}_1$	1/2	3
$(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	1st	$\mathbf{u}_1^c, \mathbf{d}_1^c, \mathbf{e}_1^c, \nu_1^c$	1/2	5
$(\mathbf{1}, \mathbf{2}, \mathbf{2})_H$	(Higgs)	$\mathbf{h}_u, \mathbf{h}_d$	0	0

Table VII. Quantum numbers of the MSSM matter introduced on the two branes.

Here we assigned all the left handed lepton doublets the same $U(1)_F$ charges to realize in our model the idea of the “democratic approach” to neutrinos [19, 20].

We could introduce the MSSM Higgs fields in the bulk or on B2 so as to avoid the notorious doublet-triplet problem. For simplicity, let us introduce them on B2 in the representation $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ of $SU(4)_c \times SU(2)_L \times SU(2)_R$. The masses of the first generation quarks and leptons are generated from the coupling $(\mathbf{4}, \mathbf{2}, \mathbf{1})(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})(\mathbf{1}, \mathbf{2}, \mathbf{2})_H$. The mass terms of the second and third generations and the mixing terms between the first and the other two generations are possible by introducing heavy vector-like $\mathbf{16}$ fields in the bulk and through interactions shown in Fig. 2 [21]. Table VIII shows the Z_2 parities and other quantum numbers of the bulk $\mathbf{16}$ fields.

Hypermultiplets	$Z_2 \times Z'_2$ parities	$U(1)_R$	$U(1)_F$
$\mathbf{16}_I$	$U_I^{c-+}, Q_I^{--}, E_I^{c-+}; D_I^{c++}, L_I^{+-}; N_I^{c+-}$	1/2	0
$\mathbf{16}_I^c$	$U_I^{+-}, Q_I^{c++}, E_I^{+-}; D_I^{--}, L_I^{c-+}; N_I^{-+}$	1/2	0
$\bar{\mathbf{16}}_I$	$\bar{U}_I^{c-+}, \bar{Q}_I^{--}, \bar{E}_I^{c-+}; \bar{D}_I^{c++}, \bar{L}_I^{+-}; \bar{N}_I^{c+-}$	1/2	0
$\bar{\mathbf{16}}_I^c$	$\bar{U}_I^{+-}, \bar{Q}_I^{c++}, \bar{E}_I^{+-}; \bar{D}_I^{--}, \bar{L}_I^{c-+}; \bar{N}_I^{-+}$	1/2	0
$\mathbf{16}_{II}$	$U_{II}^{c--}, Q_{II}^{++}, E_{II}^{c--}; D_{II}^{c+-}, L_{II}^{++}; N_{II}^{c++}$	1/2	0
$\mathbf{16}_{II}^c$	$U_{II}^{++}, Q_{II}^{c+-}, E_{II}^{++}; D_{II}^{+-}, L_{II}^{c--}; N_{II}^{--}$	1/2	0
$\bar{\mathbf{16}}_{II}$	$\bar{U}_{II}^{c--}, \bar{Q}_{II}^{++}, \bar{E}_{II}^{c--}; \bar{D}_{II}^{c+-}, \bar{L}_{II}^{++}; \bar{N}_{II}^{c++}$	1/2	0
$\bar{\mathbf{16}}_{II}^c$	$\bar{U}_{II}^{++}, \bar{Q}_{II}^{c+-}, \bar{E}_{II}^{++}; \bar{D}_{II}^{+-}, \bar{L}_{II}^{c--}; \bar{N}_{II}^{--}$	1/2	0

Table VIII. Quantum numbers assigned to the vector-like hypermultiplets.

They compose on B1 the $SU(5) \times U(1)$ multiplets $\mathbf{10}_1, \bar{\mathbf{10}}_{-1}, \bar{\mathbf{5}}_{-3}, \mathbf{5}_3, \mathbf{1}_5, \mathbf{1}_{-5}$, etc., whereas on B2 they correspond to $(\mathbf{4}, \mathbf{2}, \mathbf{1}), (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}), (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$, and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$. The $U(1)_R$ charge assignments allow for supersymmetric mass terms for them on the branes.

The effective 4D Yukawa couplings between bulk and brane fields turn out to be given by $y\sqrt{\frac{M_c}{M_*}}$ [22], where y is a coefficient of order unity, and M_c denotes the compactification scale. We assume that the effective 4D Yukawa couplings are all of order unity, and this will be justified in section 6. The resultant MSSM Yukawa couplings are

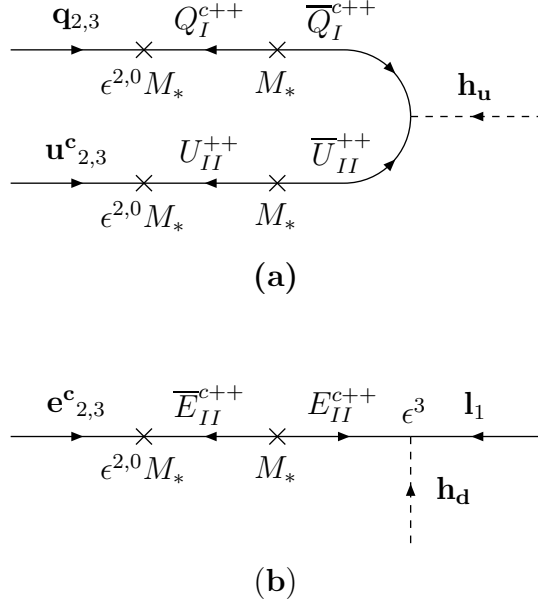


Figure 2: The effective Yukawa couplings of the second and third generation up-type quarks (a), the mixing terms between the first and the other two generations of charged leptons (b). The trilinear coupling with Higgs in (a) and (b) are present on B2. The other elements of the MSSM fermion mass matrices are also generated similarly.

$$\begin{array}{ccc}
\mathbf{u}_{2,3}^c & \mathbf{u}_{2,3}^c & \mathbf{u}_{2,3}^c \\
\mathbf{q}_1 & \left(\begin{array}{ccc} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{array} \right) & \mathbf{h}_u, \quad \mathbf{q}_1 \left(\begin{array}{ccc} \epsilon^5 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \end{array} \right) \epsilon^3 \mathbf{h}_d, \quad \mathbf{l}_1 \left(\begin{array}{ccc} \epsilon^5 & \epsilon^2 & 1 \\ \epsilon^5 & \epsilon^2 & 1 \\ \epsilon^5 & \epsilon^2 & 1 \end{array} \right) \epsilon^3 \mathbf{h}_d, \quad (45) \\
\mathbf{q}_2 & & \mathbf{q}_3 \\
\mathbf{q}_3 & & \mathbf{l}_3
\end{array}$$

where the mixing elements between the first and the last two generations are obtained after decoupling of the heavy bulk **16** fields. Diagonalization of the above matrices yields [20]

$$m_t : m_c : m_u \approx 1 : \epsilon^4 : \epsilon^8, \quad (46)$$

$$m_b : m_s : m_d \approx 1 : \epsilon^2 : \epsilon^5, \quad (47)$$

$$m_\tau : m_\mu : m_e \approx 1 : \epsilon^4 : \epsilon^8, \quad (48)$$

$$m_b \sim m_\tau \sim \frac{\epsilon^3}{\tan\beta} m_t, \quad (49)$$

where $\tan\beta \equiv \frac{\langle h_u \rangle}{\langle h_d \rangle} \sim O(1)$, and the CKM mixing angles turn out to be

$$V_{us} \sim \epsilon, \quad V_{cb} \sim \epsilon^2, \quad V_{ub} \sim \epsilon^3. \quad (50)$$

The results in Eqs. (46)–(50) are consistent with the observations.

For the neutrino sector, as mentioned, we implement the democratic scenario presented in Ref. [19]. The contributions to the solar and atmospheric neutrino mixings from the charged lepton sector in Eq.(45) are of order unity [20]. The discussion in Ref. [20] shows how bilarge mixings are realized, taking into account the contributions from the Dirac and heavy Majorana sectors. To obtain the observed solar and atmospheric neutrino masses [23, 24], one could utilize the remaining undetermined $U(1)_F$ charges of the two right handed neutrino in Table VII. Note that the third mixing angle θ_{13} is expected in this approach to be not much smaller than 0.2 or so.

Before closing this section, let us briefly discuss the μ term in the model. The $U(1)_R$ symmetry prevents a supersymmetric ‘bare’ mass term of $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ Higgs. Instead, we have the superpotential coupling

$$S(\mathbf{1}, \mathbf{2}, \mathbf{2})_H(\mathbf{1}, \mathbf{2}, \mathbf{2})_H. \quad (51)$$

The VEV $\langle S \rangle$ is zero for unbroken SUSY, but after SUSY breaking $\langle S \rangle$ becomes of order TeV, which induces the desired MSSM μ term [25].

6 Gauge Coupling Unification and Proton Decay

In this section we discuss gauge coupling unification and proton stability in the model discussed in section 4 and 5. In higher dimensional SUSY GUT models, the compactification scale would be determined from the requirement that the MSSM gauge couplings should be unified at the cutoff scale Λ ($\approx M_*$). Since the masses of the lightest colored gauge bosons such as X and Y gauge bosons in $SU(5)$ (\mathbf{Q}' , $\overline{\mathbf{Q}'}$ in Fig. 1.) are in the compactification scale, in order to discuss the proton stability, we need to analyze the renormalization effects on the gauge couplings. But unfortunately in our model there are too many unknown parameters like M_1 , M_2 , and so on.

In our paper, to simplify the analysis, we make the assumption that all mass parameters are very close to the cutoff scale Λ including the spontaneous symmetry breaking scale on the branes [17]. As discussed in section 3 and 4, the VEVs $\langle \nu^{c++} \rangle$, $\langle \bar{\nu}^{c++} \rangle$ could be constant along the extra dimension, and provide cutoff scale bulk masses to the vector and chiral multiplets charged under $U(1)_X$, belonging to $\mathbf{10}_{-4}$ and $\bar{\mathbf{10}}_4$ of $SU(5)$. We will keep only the vector and chiral multiplets of $\mathbf{24}_0$ in the renormalization group (RG) analysis of the MSSM gauge couplings. Note that the first (second and third) generation Yukawa couplings still respect $SU(4)_c \times SU(2)_L \times SU(2)_R$ ($SU(5) \times U(1)_X$).

In section 4 we introduced some vector-like pairs on B1 to make $\mathbf{5}_H$, $\bar{\mathbf{5}}_H$, $\mathbf{10}^c_H$, and $\bar{\mathbf{10}}^c_H$ in Table VI heavy (or heavier). The cutoff scale brane-localized bilinear couplings of M_b shift their masses up by just about the compactification scale M_c ($= \pi/y_c$) [26, 17], so a lot of KK modes are not decoupled. The physical masses of the fields with $(++)$ $(+-)$ parities become

$$(++) \quad : \quad 2nM_c \longrightarrow (2n+1)(1-\delta)M_c, \quad (52)$$

$$(+ -) \quad : \quad (2n+1)M_c \longrightarrow (2n+2)(1-\delta)M_c, \quad (53)$$

where δ ($\ll 1$) is proportional to M_c^2/M_b^2 . To simplify things, we set all brane-localized supersymmetric masses to be the same. Table IX shows the (shifted) mass spectrum for the bulk Higgs fields in Table V in the presence of brane localized superheavy mass terms.

\mathbf{u}^{c--}	\mathbf{e}^{c--}	\mathbf{q}^{--}	\mathbf{d}^{c++}	\mathbf{l}^{+-}
$2n+1$	$2n+1$	$2n+2$	$(2n+1)(1-\delta)$	$(2n+2)(1-\delta)$
\mathbf{u}^{+-}	\mathbf{e}^{+-}	\mathbf{q}^{c++}	\mathbf{d}^{--}	\mathbf{l}^{c--}
$(2n+2)(1-\delta)$	$(2n+2)(1-\delta)$	$(2n+1)(1-\delta)$	$2n+2$	$2n+1$
$\bar{\mathbf{u}}^{c--}$	$\bar{\mathbf{e}}^{c--}$	$\bar{\mathbf{q}}^{--}$	$\bar{\mathbf{d}}^{c++}$	$\bar{\mathbf{l}}^{+-}$
$2n+1$	$2n+1$	$2n+2$	$(2n+1)(1-\delta)$	$(2n+2)(1-\delta)$
$\bar{\mathbf{u}}^{+-}$	$\bar{\mathbf{e}}^{+-}$	$\bar{\mathbf{q}}^{c++}$	$\bar{\mathbf{d}}^{--}$	$\bar{\mathbf{l}}^{c--}$
$(2n+2)(1-\delta)$	$(2n+2)(1-\delta)$	$(2n+1)(1-\delta)$	$2n+2$	$2n+1$

Table IX. Mass spectrum of the bulk Higgs (normalized to M_c).

On the other hand, the corresponding brane fields $\bar{\mathbf{5}}_{-3}^b$, $\mathbf{5}_3^b$, $\mathbf{10}_1^b$, and $\overline{\mathbf{10}}_{-1}^b$ with unit $U(1)_R$ charges are simply decoupled due to the cutoff scale brane-localized supersymmetric mass terms.

From the electroweak to the compactification scale, only massless modes of the bulk fields and light brane fields contribute to the RG equations of the MSSM gauge couplings. On the other hand, above the compactification scale, contributions from the KK modes of the bulk fields begin to appear, so that the MSSM gauge couplings show a linear dependence on the energy scale [1, 2, 27]. Thus, above the compactification scale the evolutions are sensitive to the ultraviolet physics. However, the quantity $\Delta_i(\mu) \equiv \alpha_i^{-1}(\mu) - \alpha_1^{-1}(\mu)$ ($i = 2, 3$), displays logarithmic behavior, and can be meaningfully discussed even above the compactification scale [1, 2]. In our model we have

$$\Delta_3(m_Z) = \frac{1}{2\pi} \left[-\frac{48}{5} \ln \frac{\Lambda}{m_Z} - 6 \sum_{n=0}^N \ln \frac{\Lambda}{(2n+2)M_c} + 6 \sum_{n=0}^N \ln \frac{\Lambda}{(2n+1)M_c} \right], \quad (54)$$

$$\Delta_2(m_Z) = \frac{1}{2\pi} \left[-\frac{28}{5} \ln \frac{\Lambda}{m_Z} - 4 \sum_{n=0}^N \ln \frac{\Lambda}{(2n+2)M_c} + 4 \sum_{n=0}^N \ln \frac{\Lambda}{(2n+1)M_c} \right], \quad (55)$$

with $\Lambda \gtrsim (2N+2)M_c$. Here we set $\alpha_3 = \alpha_2 = \alpha_1$ at $\mu = \Lambda$. Note that the δ dependences in Eqs. (54) and (55) exactly cancel out. The beta function coefficients of the first terms in Eqs. (54) and (55) are the same as in the MSSM case. They result from contributions from the zero modes in $\mathbf{24}_0$, and the brane matter fields of Table VII. Note that the vector-like superfields shown in Table VIII would draw the MSSM gauge couplings much larger values upto the cutoff scale. But they do not affect $\Delta_3(\mu)$ and $\Delta_2(\mu)$, because the superfields with the same $Z_2 \times Z'_2$ parities in Table VIII compose complete $\mathbf{16}$, $\overline{\mathbf{16}}$ of $SO(10)$.

The linear combination $-[2\Delta_3(m_Z) + 3\Delta_2(m_Z)]$ gives

$$5\alpha_1^{-1}(m_Z) - 3\alpha_2^{-1}(m_Z) - 2\alpha_3^{-1}(m_Z) = \frac{1}{2\pi} \left[36 \ln \frac{\Lambda}{m_Z} - 24 \sum_{n=0}^N \frac{2n+2}{2n+1} \right], \quad (56)$$

which interestingly coincides with the result in Ref. [1]. Comparison with the corre-

sponding linear combination in the usual 4D MSSM leads to

$$\ln \frac{M_c}{m_Z} = \ln \frac{M_U}{m_Z} + \frac{2}{3} \sum_{n=0}^N \ln \frac{2n+2}{2n+1} - \ln(2N+2) , \quad (57)$$

where M_U lies in the range $1 \times 10^{15} \text{ GeV} \lesssim M_U \lesssim 3 \times 10^{16} \text{ GeV}$ from the experimental values of the gauge couplings. In section 5 we assumed that the effective Yukawa coupling $y\sqrt{\frac{M_c}{M_*}} (\approx y\sqrt{\frac{M_c}{\Lambda}})$ is $O(1)$, where $y \sim O(1)$. If we take $\Lambda \approx 10M_c$ ($N = 4$) as in Ref. [1], which is also consistent with our assumption, M_c is restricted by

$$3 \times 10^{15} \text{ GeV} \lesssim M_c \lesssim 8 \times 10^{15} \text{ GeV} . \quad (58)$$

Hence, the compactification scale can be high enough to fulfill the bound $M_c \gtrsim 5 \times 10^{15} \text{ GeV}$ arising from proton decay experiments and the constraints on the masses of the X, Y gauge bosons in $SU(5)$ [28, 1, 29]. Note that improvements by an order of magnitude of proton decay experiments would severely constrain our model or find proton decay! Finally note that baryon number violating dimension five operators are eliminated by $U(1)_R$ [25].

Another useful linear combination $7\Delta_3(m_Z) - 12\Delta_2(m_Z)$ or

$$5\alpha_1^{-1}(m_Z) - 12\alpha_2^{-1}(m_Z) + 7\alpha_3^{-1}(m_Z) = \frac{1}{2\pi} \left[-6 \sum_{n=0}^N \ln \frac{2n+2}{2n+1} \right] \quad (59)$$

provides a bound on N , but it is rather weak due to the experimental uncertainty of $\alpha_3(m_Z)$ [1]. The experimental data prefers a positive value on the right hand side of Eq. (59). Let us discuss how this could arise from threshold corrections with an example. In contrast to 4D $SO(10)$, $SU(4)_c \times SU(2)_L \times SU(2)_R$ as well as $[SU(4)_c \times SU(2)_L \times SU(2)_R]/Z_2$ can be embedded in 5D $SO(10)$ compactified on $S^1/(Z_2 \times Z'_2)$. Hence, superfields such $(\mathbf{4}, \mathbf{1}, \mathbf{1}) (= (\mathbf{3}, \mathbf{1})_{1/6} + (\mathbf{1}, \mathbf{1})_{-1/2})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}) (= (\bar{\mathbf{3}}, \mathbf{1})_{-1/6} + (\mathbf{1}, \mathbf{1})_{1/2})$ carrying $U(1)_R$ charges of $1/2$ and their supersymmetric mass terms may be introduced on B2. While they leave intact Eq. (56), each pair yields an additional positive contribution to Eq. (59),

$$+ \frac{1}{2\pi} \times \left[9 \ln \frac{\Lambda}{m_4} \right] , \quad (60)$$

where m_4 denotes the supersymmetric mass of $(\mathbf{4}, \mathbf{1}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1})$.⁴ This can be employed to flip the sign of the right hand side of Eq. (59) without changing M_c and Λ . Thus, by introducing two pairs of such fields with mass $\sim \Lambda/10$, $\alpha_3(m_Z)$ can be brought closer to the experimental value (0.117 ± 0.002) [30] than the MSSM prediction. Alternatively, we can achieve the same result with two $(\mathbf{6}, \mathbf{1}, \mathbf{1})$'s ($= (\mathbf{3}, \mathbf{1})_{-1/3} + (\bar{\mathbf{3}}, \mathbf{1})_{1/3}$) of $[SU(4)_c \times SU(2)_L \times SU(2)_R]/Z_2$ with suitable $U(1)_R$ charges.⁵

We have confined our discussion of gauge coupling unification to the model with gauge groups $SU(5) \times U(1)_X - SU(4)_c \times SU(2)_L \times SU(2)_R$. We can expect that an analogous discussion on gauge coupling unification for the $SU(5) \times U(1)_X - SU(5)' \times U(1)'_X$ model can be carried out by introducing heavy vector-like pair(s) of chiral fields. For instance, we can introduce $\mathbf{10}'_1 (= (\bar{\mathbf{3}}, \mathbf{1})_{1/3} + (\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{1}, \mathbf{1})_0)$ and $\bar{\mathbf{10}}'_{-1}$, or $\mathbf{1}'_5 (= (\mathbf{1}, \mathbf{1})_1)$ and $\bar{\mathbf{1}}'_{-5}$ on the $SU(5)' \times U(1)'_X$ brane. While the $\mathbf{10}'_1, \bar{\mathbf{10}}'_{-1}$ pair shifts up α_2 and α_3 relative to α_1 , the $\mathbf{1}'_5, \bar{\mathbf{1}}'_{-5}$ pair only contributes to α_1 near the cut-off scale.⁶ Thus, depending on details of the model, it seems that one can always achieve gauge coupling unification using such pairs.

7 Conclusion

We have considered a variety of symmetry breakings obtained from compactifying 5D $SO(10)$ on $S^1/(Z_2 \times Z'_2)$. In particular, the residual symmetry after compactification is $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$. We have seen how the MSSM can be realized at low energies after spontaneous breaking of $U(1)_X$. We have presented the implication of one particular example, in which the Higgs breaking scale of $U(1)_X$ is comparable to the cutoff scale. Thus, effectively we have the breaking $SO(10) \rightarrow SU(5) \rightarrow$

⁴The contributions from $(\mathbf{3}, \mathbf{1})_{1/6}, (\mathbf{1}, \mathbf{1})_{-1/2}, (\bar{\mathbf{3}}, \mathbf{1})_{-1/6}, (\mathbf{1}, \mathbf{1})_{1/2}$ to the evolutions of the three MSSM gauge couplings $\alpha_1^{-1}, \alpha_2^{-1}$ and α_3^{-1} are respectively given by $b_1 = \frac{3}{5}[(\frac{1}{6})^2 \times 3 \times 2 + (\frac{1}{2})^2 \times 2] = \frac{2}{5}$, $b_2 = 0$, $b_3 = \frac{1}{2}[1 \times 2] = 1$ (upto a factor of $\frac{1}{2\pi} \ln \frac{\Lambda}{\mu}$). Hence, their contribution to $5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}$ vanishes, while they yield Eq. (60).

⁵The contributions from $(\mathbf{3}, \mathbf{1})_{-1/3}, (\bar{\mathbf{3}}, \mathbf{1})_{1/3}$ to the evolution of $\alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1}$ are respectively $b_1 = \frac{3}{5}[(\frac{1}{3})^2 \times 3 \times 2] = \frac{2}{5}$, $b_2 = 0$, $b_3 = \frac{1}{2}[1 \times 2] = 1$ (upto a factor of $\frac{1}{2\pi} \ln \frac{\Lambda}{\mu}$).

⁶ $\mathbf{10}'_1$ and $\bar{\mathbf{10}}'_{-1}$ give $b_1 = \frac{3}{5}[(\frac{1}{3})^2 \times 3 \times 2 + (\frac{1}{6})^2 \times 6 \times 2] = \frac{3}{5}$, $b_2 = \frac{1}{2}[3 \times 2] = 3$, $b_3 = \frac{1}{2}[(1+2) \times 2] = 3$. Hence, $b_1 < b_2 = b_3$. On the other hand, $\mathbf{1}'_5$ and $\bar{\mathbf{1}}'_{-5}$ just give $b_1 = \frac{3}{5}[1^2 \times 2] = \frac{6}{5}$, $b_2 = b_3 = 0$.

MSSM, for which we study the implications for fermion masses and mixings, gauge coupling unification and proton decay.

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